Description of multipole asymmetries with the Buda-Lund hydrodynamical model

Máté Csanád, Sándor Lökös, Boris Tomášik and Tamás Csörgő

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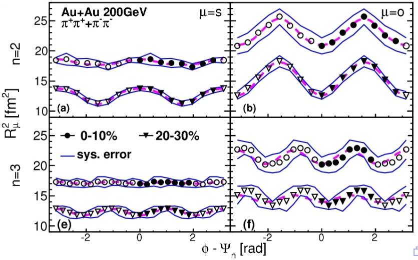
based on the WPCF presentation

Introduction

- QGP behaves like perfect fluid $\rightarrow$ hydro description
- Finite number of nucleons $\rightarrow$ generalized geometry is nescessary
- Generalize the space-time and the velocity field distribution
- Higher order flows can be investigated
- HBT radii have $\cos(n\phi)$ dependences in the respective reaction plane
- These can be studied experimentally:

  Nucl.Phys. A904-905 (2013) 439c-442c

  Phys.Rev.Lett. 112 (2014) 22, 222301



The Buda-Lund model

Phys.Rev. C54 (1996) 1390 and Nucl.Phys. A742 (2004) 80-94

- Hydro-model: $S(x,p) = \frac{g}{(2\pi)^3} \frac{p^\nu d^4\Sigma_\nu(x)}{B(x,p)+s_q}$ where $B(x,p) = \exp\left[\frac{p^\nu u_\nu(x) - \mu(x)}{T(x)}\right]$ is the Boltzmann phase-space distribution and the $p^\nu d^4\Sigma_\nu(x) = p^\nu u_\nu H(\tau) d^4x$
- In this case

$$H(\tau) = \frac{1}{\sqrt{(2\pi\Delta\tau^2)}} e^{-\frac{(\tau-\tau_0)^2}{2\Delta\tau^2}}$$

- Later I will assume $\Delta\tau^2 \to 0$ so $H(\tau) = \delta(\tau - \tau_0)$.
- There is a Gaussian-profile in fugacity and temperature profile with $a^2 = \frac{T_0 - T_s}{T_s}$

$$\frac{\mu(x)}{T(x)} = \frac{\mu_0}{T_0} - bs \qquad \frac{1}{T(x)} = \frac{1}{T_0}\left(1 + a^2 s\right)$$

The Buda-Lund model

- Spatial elliptical asymmetry is ensured by the scaling variable

$$s = \frac{r_x^2}{2X^2} + \frac{r_y^2}{2Y^2} + \frac{r_z^2}{2Z^2} \quad \rightarrow \quad \frac{r^2}{2R^2}\left(1 + \epsilon_2 \cos(2\phi)\right) + \frac{r_z^2}{2Z^2}$$

- The asymmetry in the velocity field is also elliptical

$$u_\mu = \left(\gamma, r_x \frac{\dot{X}}{X}, r_y \frac{\dot{Y}}{Y}, r_z \frac{\dot{Z}}{Z}\right) \rightarrow$$
$$\rightarrow \left(\gamma, rH(1 + \chi_2)\cos(\phi), rH(1 - \chi_2)\sin(\phi), H_z r_z\right)$$

With all of these definition an analytic model can be derived from a saddle-point-approximate form of source function

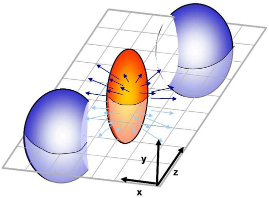
$$S(x,k)d^4x = \frac{g}{(2\pi)^3} \frac{p_\mu u^\mu(x_s)\delta(\tau - \tau_0)}{B(x_s, p)} e^{R_{\mu\nu}^{-2}(x-x_s)^\mu (x-x_s)^\nu} d^4x$$

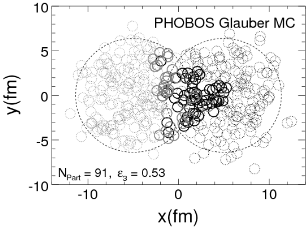
Generalization of the model I.

- The spatial asymmetry is described by the scaling variable
- General n-pole spatial asymmetry (elliptical case: $n = 2$):

$$s = \frac{r^2}{2R^2}\left(1 + \sum_n \epsilon_n \cos(n(\phi - \Psi_n))\right) + \frac{r_z^2}{2Z^2}$$

- Ψ_n is the angle of the n-th order reaction plane



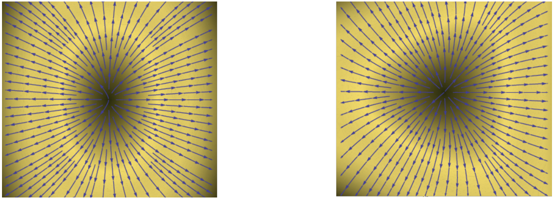


Generalization of the model II.

- Derive the velocity field from a potential: $u_\mu = \gamma(1, \partial_x\Phi, \partial_y\Phi, \partial_z\Phi)$
- General n-pole asymmetrical potential (elliptical case: $n = 2$):

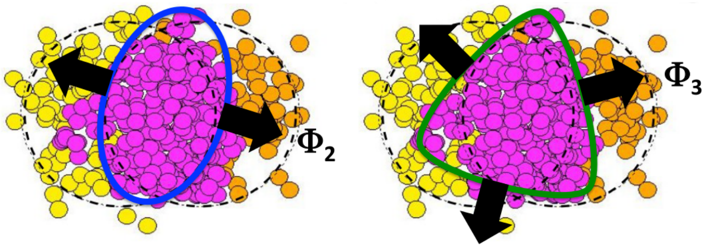
$$\Phi = Hr^2\left(1 + \sum_n \chi_n \cos(n(\phi - \Psi_n))\right) + H_z r_z^2$$

- There is multipole solution: Phys.Rev.C90,054911 (2014) based on HeavyIonPhys.A21:73-84,2004



Observables at freeze-out

- Invariant transverse momentum distribution, flows, azimuthally sensitive HBT radii
- All asymmetries are investigated in their respective reaction plane
- Rotate the system to the second / third order plane and average on the angle of the third / second order plane
- The proper parameters can be set to zero



Averaging on event planes

Very simply model: $S(x) = e^{-s}$

- Let the $s = \frac{r^2}{R^2}(1 + \epsilon_2 \cos(2\phi - \Psi_2) + \epsilon_3 \cos(3\phi - \Psi_3)) + \frac{r_z^2}{Z^2}$
- If we rotate the system to the second order event plane:

$$s = \frac{r^2}{R^2}(1 + \epsilon_2 \cos(2\phi) + \epsilon_3 \cos(3\phi - \Delta\Psi_{2,3})) + \frac{r_z^2}{Z^2}$$

- If we rotate the system to the third order event plane:

$$s = \frac{r^2}{R^2}(1 + \epsilon_2 \cos(2\phi + \Delta\Psi_{2,3}) + \epsilon_3 \cos(3\phi)) + \frac{r_z^2}{Z^2}$$

where $\Delta\Psi_{2,3} = \Psi_3 - \Psi_2$.

Averaging on event planes

There is two possibilites:

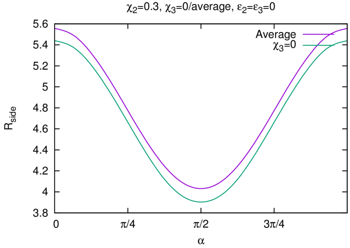
- Set to zero the parameter directly

$$\int d\phi S(x) = 2\pi e^{-\frac{r^2}{R^2}} I_0\left(\epsilon_2 \frac{r^2}{R^2}\right)$$

- Averaging on $\Delta\Psi_{2,3} = \Psi_3 - \Psi_2$

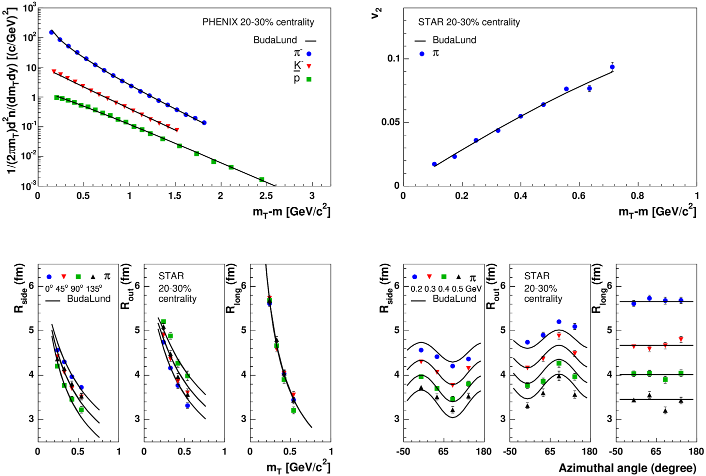
$$\int d\phi S_{av}(x) = 2\pi e^{-\frac{r^2}{R^2}} I_0\left(\epsilon_2 \frac{r^2}{R^2}\right) I_0\left(\epsilon_3 \frac{r^2}{R^2}\right)$$

- There is a difference in the "real case" too



Earlier results

Fits with elliptical Buda Lund model: Eur.Phys.J. A47 (2011) 58-66



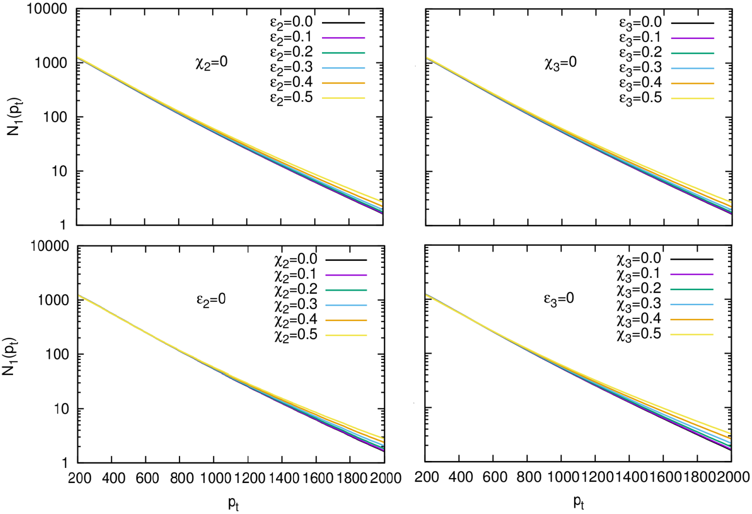
Value of the parameters

Meaning	Sign	Value
Mass of the particle	m	140 MeV
Freeze-out time	τ_0	7 fm/c
Freeze-out temperature	T_0	170 MeV
Temperature-asymmetry parameter	a^2	0.3
Spatial slope parameter	b	-0.1
Transverse size of the source	R	10 fm
Longitudinal size of the source	Z	15 fm
Velocity-space transverse size	H	10 c/fm
Velocity-space longitudinal size	H_z	16 c/fm
Elliptical spatial asymmetry parameter	ϵ_2	0.0
Triangular spatial asymmetry parameter	ϵ_3	0.0
Elliptical velocity-field asymmetry parameter	χ_2	0.0
Triangular velocity-field asymmetry parameter	χ_3	0.0

Usually one anisotropy parameter is varied, and the others are kept zero

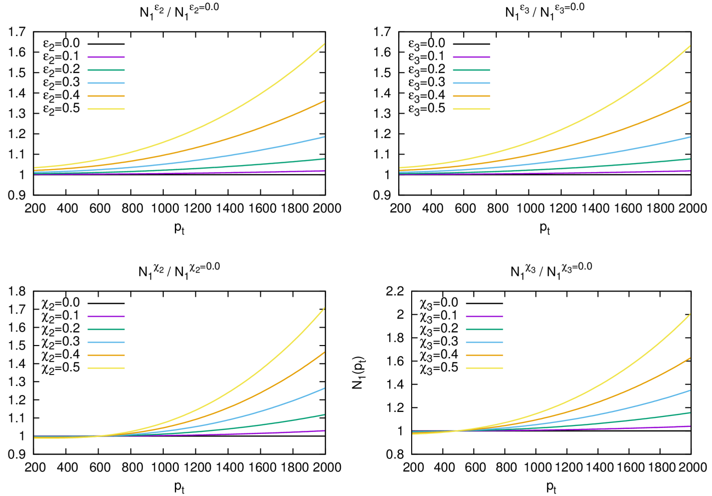
Invariant momentum distribution

Significant change could be at high p_t, the log slope is not affected strongly



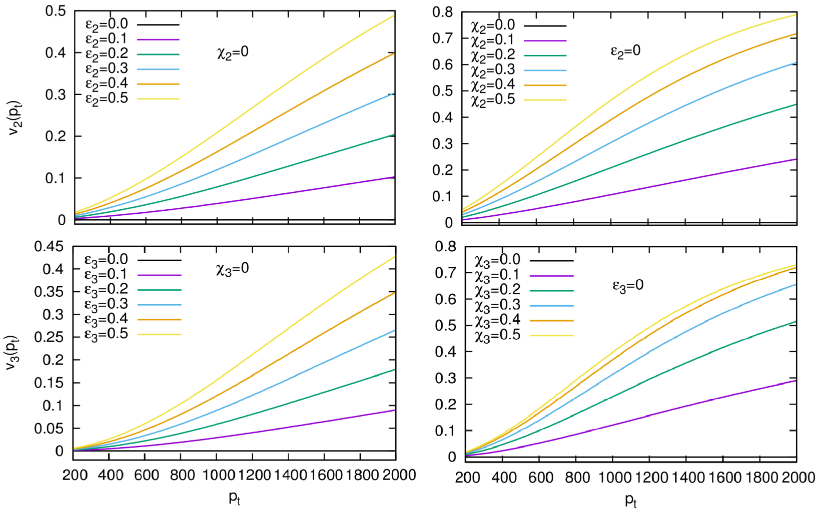
About the spectra

Plot the N_1 with non zero coefficient divide by N_1 with zero coefficient



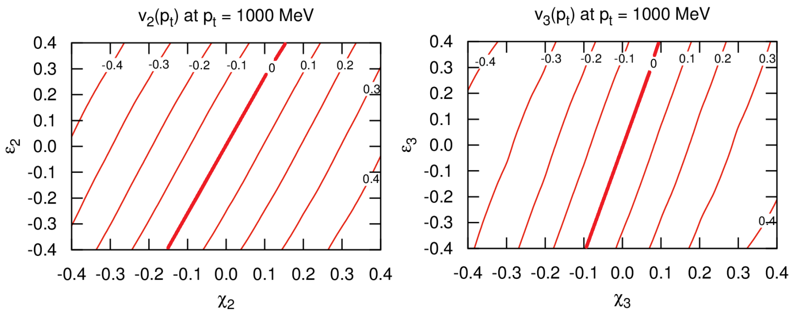
Flows

Elliptic and triangular flows are affected by their own asymmetry parameters



Mixing of parameters

- The parameters affect the flows together
- The generalization of velocity field is nescessary



HBT radii

- Calculate in the *out* – *side* – *long* system

$$R_{\text{out}}^2 = \langle r_{\text{out}}^2 \rangle - \langle r_{\text{out}} \rangle^2 \text{ and } R_{\text{side}}^2 = \langle r_{\text{side}}^2 \rangle - \langle r_{\text{side}} \rangle^2$$

  where $r_{\text{out}} = r\cos(\phi - \alpha) - \beta_t t$ and $r_{\text{side}} = r\sin(\phi - \alpha)$

  → C. J. Plumberg, C. Shen, U. W. Heinz Phys.Rev. C88 (2013) 044914

- There can be higher order parts

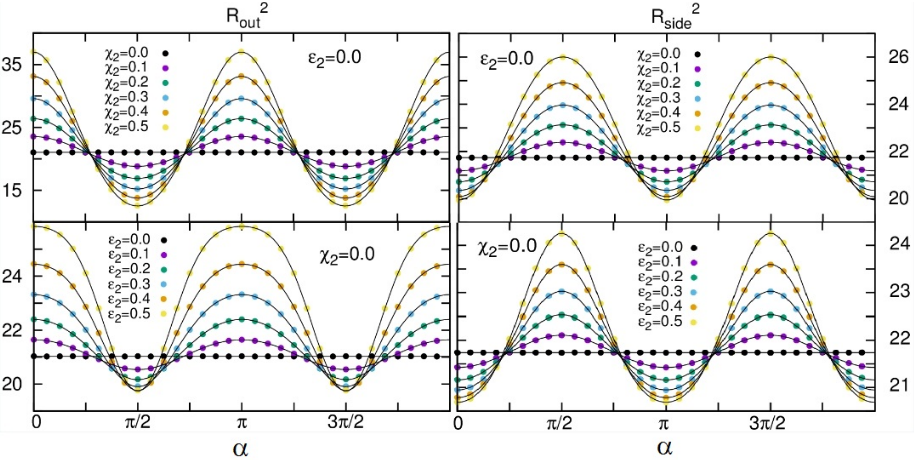
  → B. Tomášik and U. A. Wiedemann, in *QGP3*, pp. 715–777.

- We use the following parameterization in
  - elliptical case:
    $R_{\text{out}}^2 = R_{\text{out},0}^2 + R_{\text{out},2}^2\cos(2\alpha) + +R_{\text{out},4}^2\cos(4\alpha) + R_{\text{out},6}^2\cos(6\alpha)$
  - triangular case:
    $R_{\text{out}}^2 = R_{\text{out},0}^2 + R_{\text{out},3}^2\cos(3\alpha) + R_{\text{out},6}^2\cos(6\alpha) + R_{\text{out},9}^2\cos(9\alpha)$

- Similar to the R_{side}^2

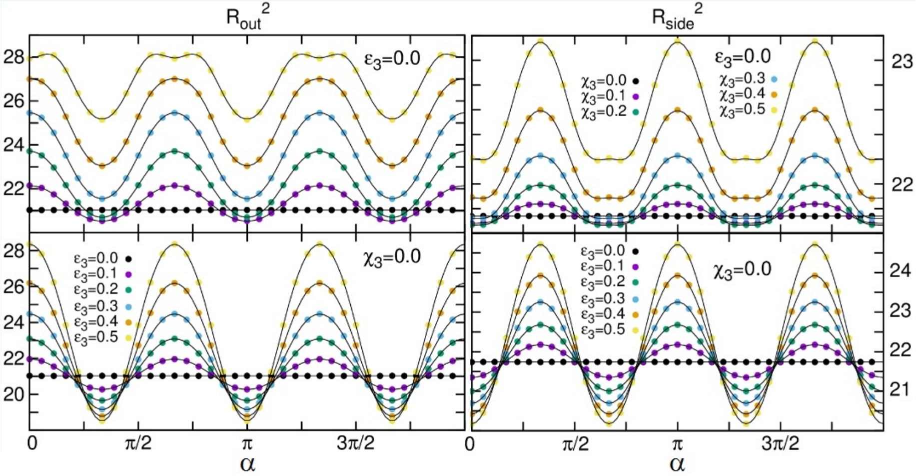
Results of the parametrization – Second order case

This case already have investigated: Eur.Phys.J.A37:111-119,2008
Mainly $\cos(2\phi)$ behavior but higher order oscillations are also present



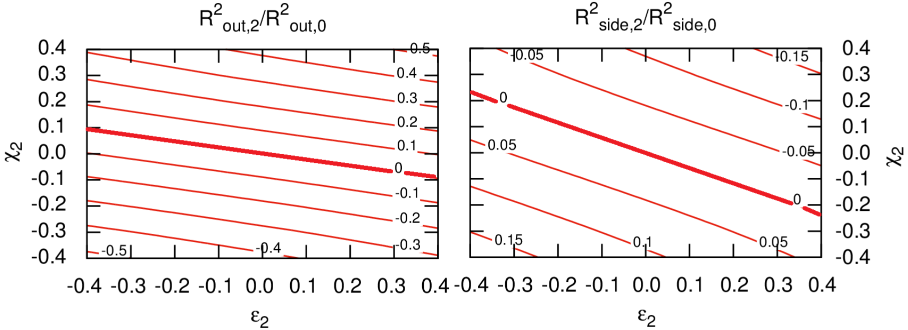
Results of the parametrization – Third order case

Mainly $\cos(3\phi)$ behavior but higher order oscillations are also present

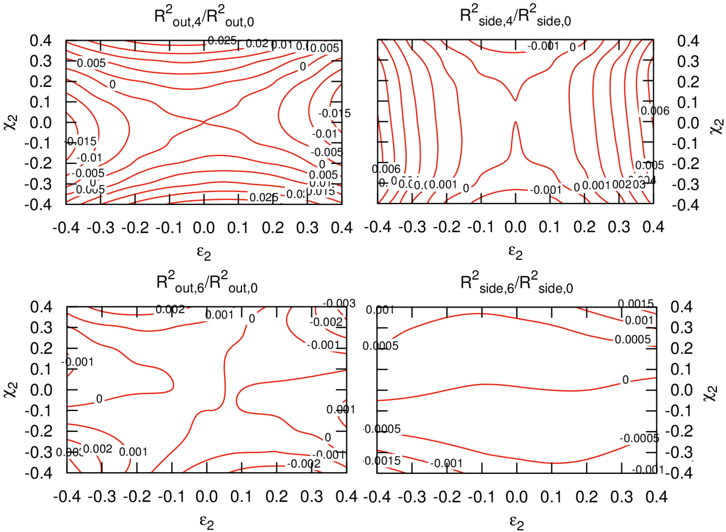


Mixing of the parameters

The dependence of the amplitudes of the R^2_{out} and R^2_{side} in the second order case

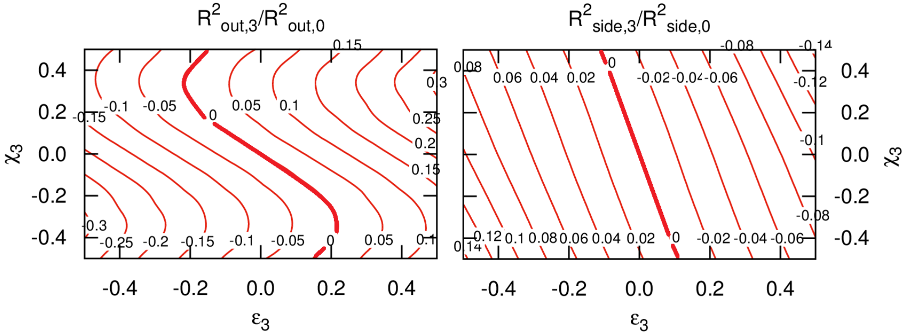


Second order:



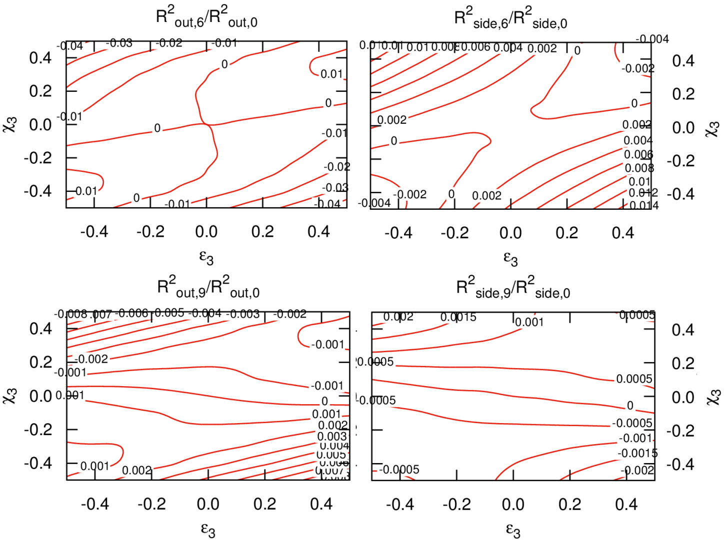
Mixing of the parameters

The dependence of the amplitudes of the R^2_{out} and R^2_{side} in the third order case



Higher order amplitudes

Third order:



Conclusions

- Generalization of u^μ and s with $u^\mu \partial_\mu s = 0$ kept valid
- The dynamics of the parameters can be obtained
- Absolute value of the azimuthal HBT radii depend on asymmetries
- Higher order oscillation can be observed in HBT radii
- The spatial and velocity field anisotropies both influence the v_n coefficient and the HBT radii
- The asymmetry parameters can be disentangle from the flows and the amplitudes

Thank you for your attention!