Higher order anisotropies in the Buda-Lund model: Disentangling flow and density field anisotropies

Máté Csanád, Sándor Lökös, Boris Tomášik and Tamás Csörgő

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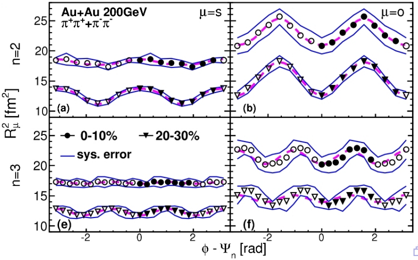
- sQGP behaves like perfect fluid $\rightarrow$ hydro description
- Finite number of nucleons $\rightarrow$ generalized geometry is necessary
- Generalize the space-time and the velocity field distribution
- Multipole in space-time solution: PRC90,054911 $\rightarrow$ higher order flows
- asHBT have $\cos(n\phi)$ dependences $\rightarrow$ generalized velocity field needed
- These can be studied experimentally:

  Nucl.Phys. A904-905 (2013) 439c-442c

  Phys.Rev.Lett. 112 (2014) 22, 222301



The Buda-Lund model

Phys.Rev. C54 (1996) 1390 and Nucl.Phys. A742 (2004) 80-94

- Hydro-model: $S(x,p) = \frac{g}{(2\pi)^3} \frac{p^\nu d^4\Sigma_\nu(x)}{B(x,p)+s_q}$ where $B(x,p) = \exp\left[\frac{p^\nu u_\nu(x)-\mu(x)}{T(x)}\right]$ is the Boltzmann phase-space distribution and the $p^\nu d^4\Sigma_\nu(x) = p^\nu u_\nu \delta(\tau-\tau_0) d^4x$
- Spatial elliptical asymmetry is ensured by the scaling variable

$$s = \frac{r_x^2}{2X^2} + \frac{r_y^2}{2Y^2} + \frac{r_z^2}{2Z^2} \to \frac{r^2}{2R^2}(1+\epsilon_2\cos(2\phi)) + \frac{r_z^2}{2Z^2}$$

- The asymmetry in the velocity field is also elliptical

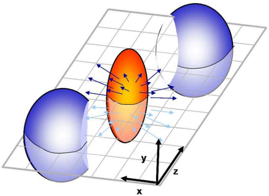
$$u_\mu = \left(\gamma, r_x\frac{\dot{X}}{X}, r_y\frac{\dot{Y}}{Y}, r_z\frac{\dot{Z}}{Z}\right) \to (\gamma, rH(1+\chi_2)\cos(\phi), rH(1-\chi_2)\sin(\phi), H_z r_z)$$

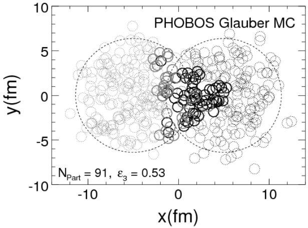
Generalization of the model I. Eur. Phys. J. A (2016) 52: 311

- The spatial asymmetry is described by the scaling variable
- General n-pole spatial asymmetry (elliptical case: $n = 2$):

$$s = \frac{r^2}{2R^2}\left(1 + \sum_n \epsilon_n \cos(n(\phi - \Psi_n))\right) + \frac{r_z^2}{2Z^2}$$

- Ψ_n is the angle of the n-th order reaction plane

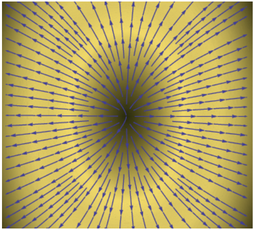


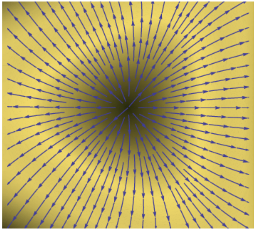


Generalization of the model II. Eur. Phys. J. A (2016) 52: 311

- Derive the velocity field from a potential: $u_\mu = \gamma(1, \partial_x \Phi, \partial_y \Phi, \partial_z \Phi)$
- General n-pole asymmetrical potential (elliptical case: $n = 2$):

$$\Phi = H\frac{r^2}{2}\left(1 + \sum_n \chi_n \cos(n(\phi - \Psi_n))\right) + H_z \frac{r_z^2}{2}$$





General aspects Eur. Phys. J. A (2016) 52: 311

There is multipole solution:
Phys.Rev.C90,054911(2014) based on HeavyIonPhys.A21:73-84(2004)
In our case $u^\mu \partial_\mu s = 0$ can be fulfilled :

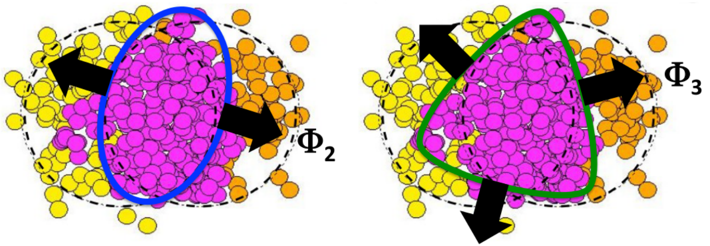
- in $\mathcal{O}(\epsilon_n)$ and $\mathcal{O}(\chi_n)$ if $\dot{\epsilon}_n = -2\frac{\dot{R}}{R}\chi_n$ and $\frac{\dot{R}}{R} = H$
- in any order $\frac{\dot{R}}{R} = H\left(1 + \frac{1}{2}\sum_{n=1}^{\infty} \epsilon_n \chi_n \left(1 + \frac{n^2}{4}\right)\right)$

Or for any $k > 0$

$$\dot{\epsilon}_k = 2H\chi_k - 2\left(\frac{\dot{R}}{R} - H\right)\epsilon_k + \\ + H\sum_{n=1}^{\infty} \epsilon_n \chi_{n+k}\left(1 + \frac{n(n+k)}{4}\right) + \\ + H\sum_{\substack{n=1 \\ n\neq k}}^{\infty} \epsilon_n \chi_{|n-k|}\left(1 + \frac{n(n-k)}{4}\right)$$

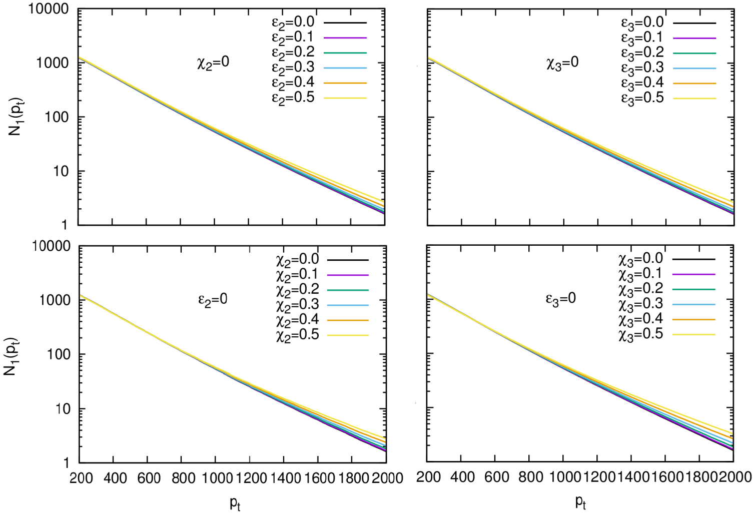
Observables at freeze-out

- Invariant transverse momentum distribution, flows, azimuthally sensitive HBT radii
- All asymmetries are investigated in their respective reaction plane
- Rotate the system to the second / third order plane and average on the angle of the third / second order plane
- The proper parameters can be set to zero to avoid the averaging



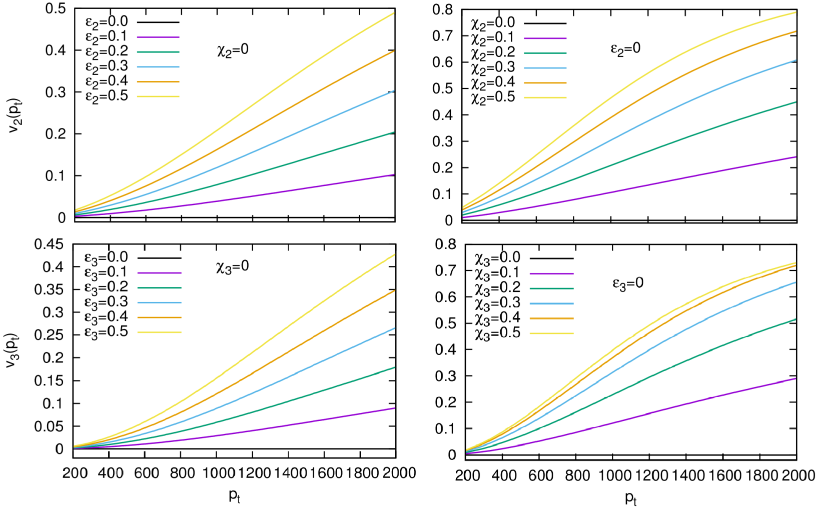
Invariant momentum distribution

Significant change could be at high p_t, the log slope is not affected strongly



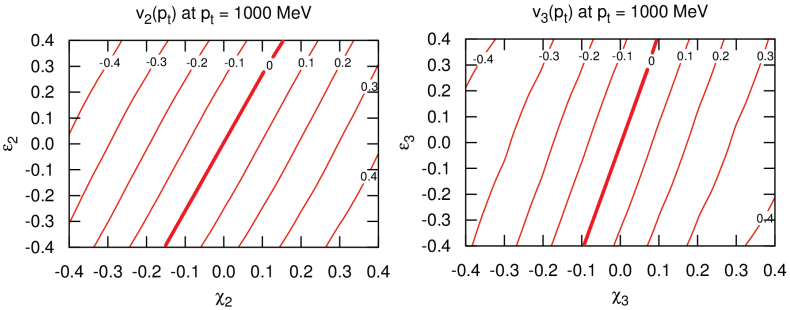
Flows

Elliptic and triangular flows are affected by their own asymmetry parameters



Mixing of parameters

- The parameters affect the flows together
- The generalization of the velocity field is necessary



HBT radii

- Calculate in the *out* – *side* – *long* system

$$R_{\text{out}}^2 = \langle r_{\text{out}}^2 \rangle - \langle r_{\text{out}} \rangle^2 \text{ and } R_{\text{side}}^2 = \langle r_{\text{side}}^2 \rangle - \langle r_{\text{side}} \rangle^2$$

  where $r_{\text{out}} = r\cos(\phi - \alpha) - \beta_t t$ and $r_{\text{side}} = r\sin(\phi - \alpha)$

  → C. J. Plumberg, C. Shen, U. W. Heinz Phys.Rev. C88 (2013) 044914

- There can be higher order parts

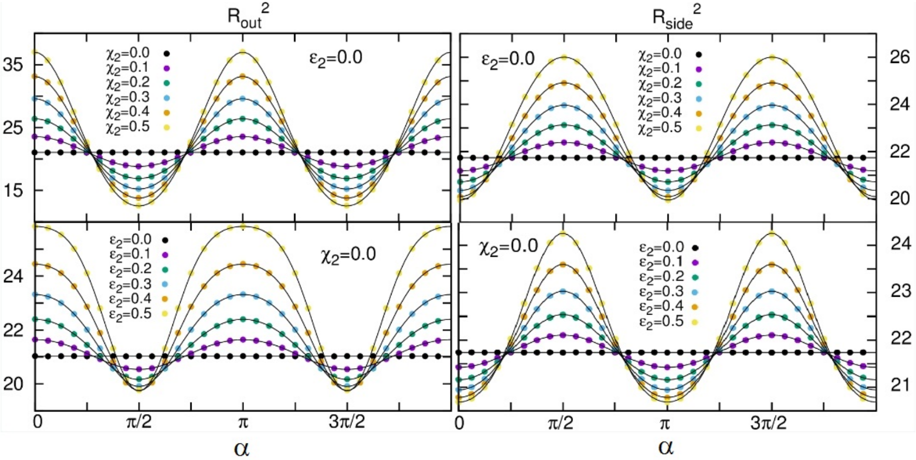
  → B. Tomášik and U. A. Wiedemann, in *QGP3*, pp. 715–777.

- We use the following parametrisation in
  - elliptical case:
    $R_{\text{out}}^2 = R_{\text{out},0}^2 + R_{\text{out},2}^2 \cos(2\alpha) + +R_{\text{out},4}^2 \cos(4\alpha) + R_{\text{out},6}^2 \cos(6\alpha)$
  - triangular case:
    $R_{\text{out}}^2 = R_{\text{out},0}^2 + R_{\text{out},3}^2 \cos(3\alpha) + R_{\text{out},6}^2 \cos(6\alpha) + R_{\text{out},9}^2 \cos(9\alpha)$
- Similar to the R_{side}^2

Results of the parametrization – Second order case

This case already have investigated: Eur.Phys.J.A37:111-119,2008

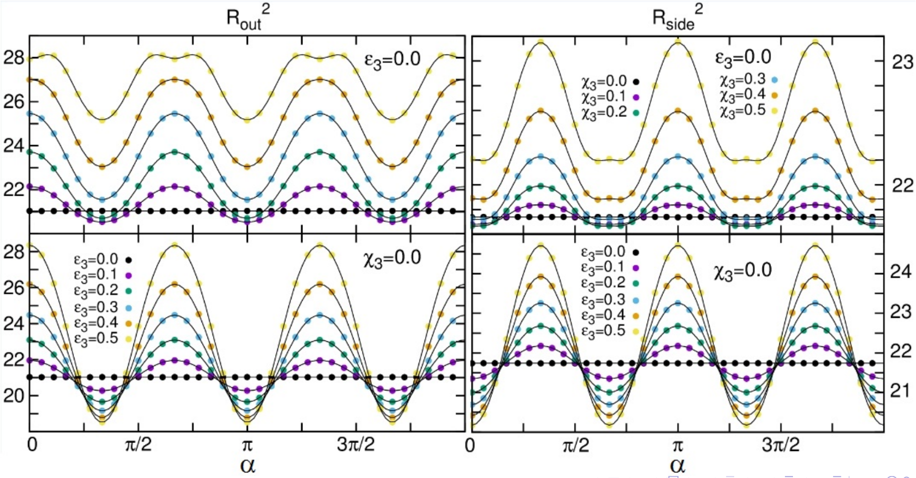
Mainly $\cos(2\phi)$ behaviour but higher order oscillations are also present



Results of the parametrization – Third order case

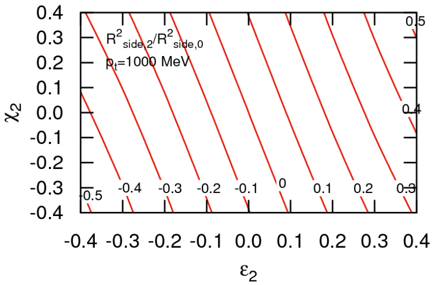
For details see: Eur. Phys. J. A (2016) 52: 311

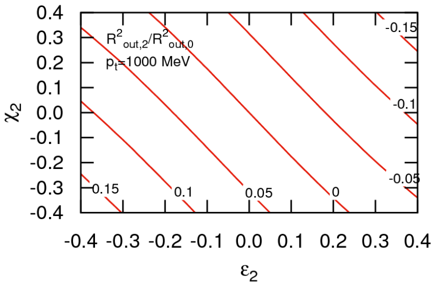
Mainly $\cos(3\phi)$ behaviour but higher order oscillations are also present



Mixing of the parameters

The dependence of the amplitudes of the R^2_{out} and R^2_{side} in the second order case





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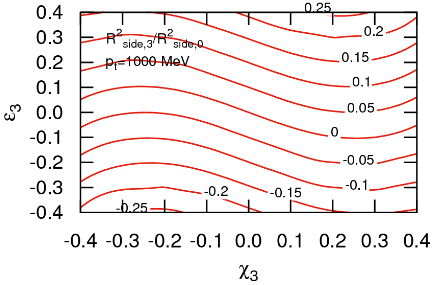
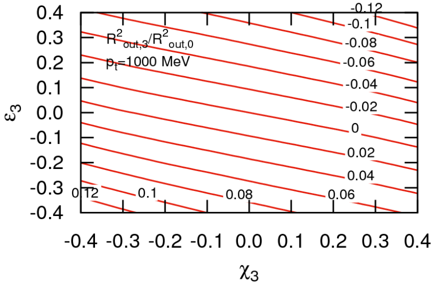
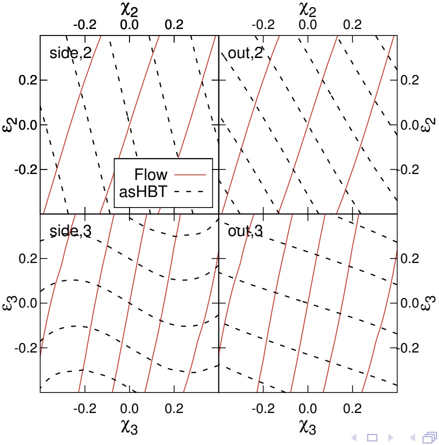




Disentangling

How to disentangle the parameters of the flow and the asHBT radii?
With a simultaneous measurements of these two observables



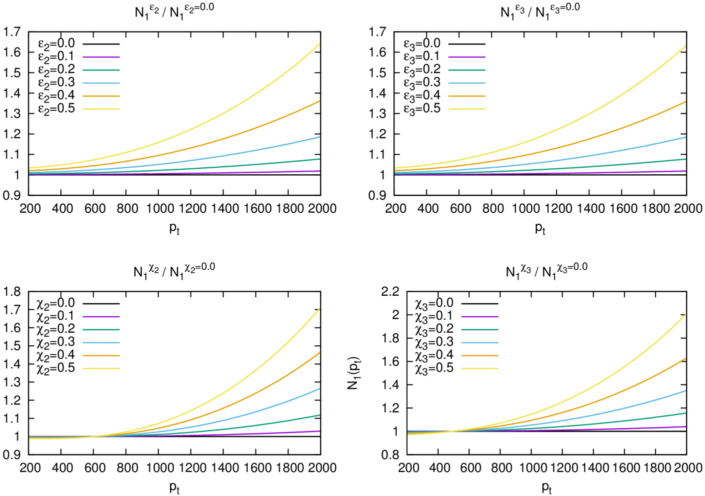
Conclusions

- Generalization of u^μ and s with $u^\mu \partial_\mu s = 0$ kept valid
- Higher order flows and azimuthally sensitive HBT radii can be derived
- Absolute value of the azimuthal HBT radii depend on asymmetries
- Higher order oscillation can be observed in HBT radii
- The spatial and velocity field anisotropies both influence the v_n coefficient and the HBT radii
- The asymmetry parameters can be disentangle from the flows and the amplitudes

Thank you for your attention!

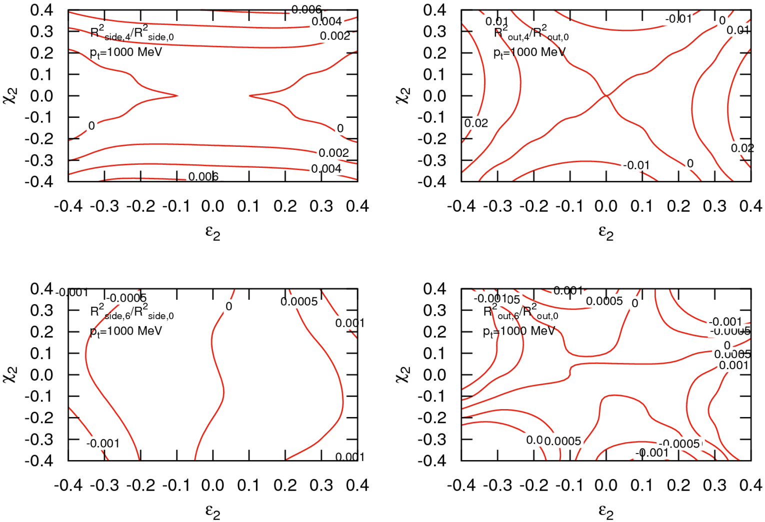
Backup slides – About the spectra

Plot the N_1 with non zero coefficient divide by N_1 with zero coefficient



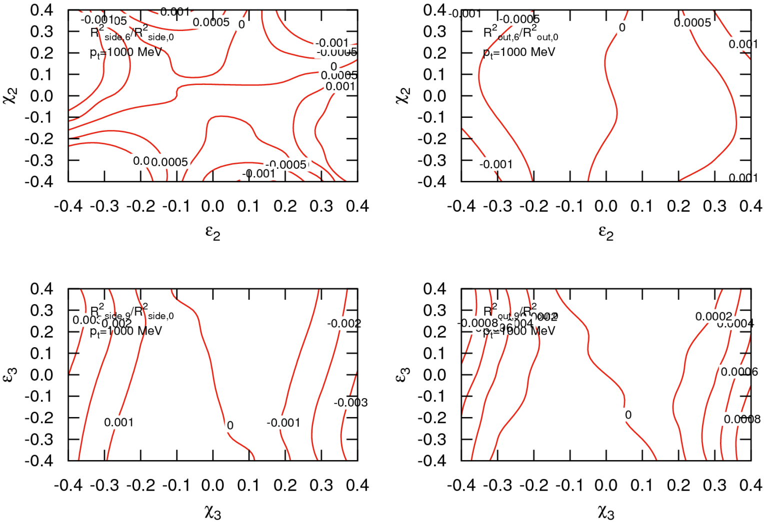
Backup slides – Higher order amplitudes

Second order:



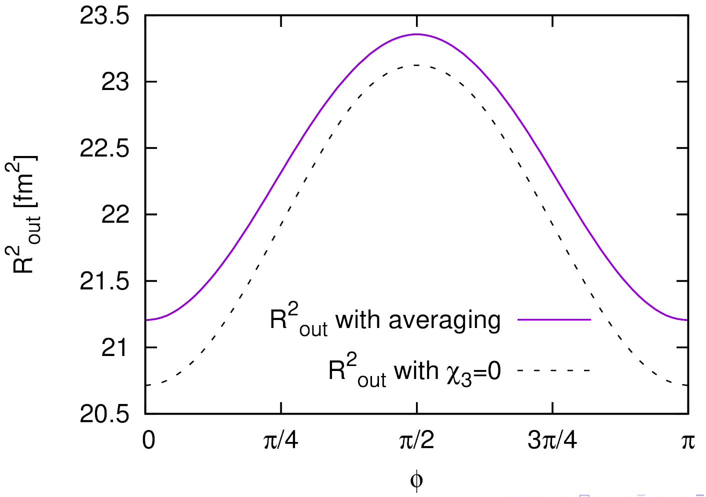
Backup slides – Higher order amplitudes

Third order:



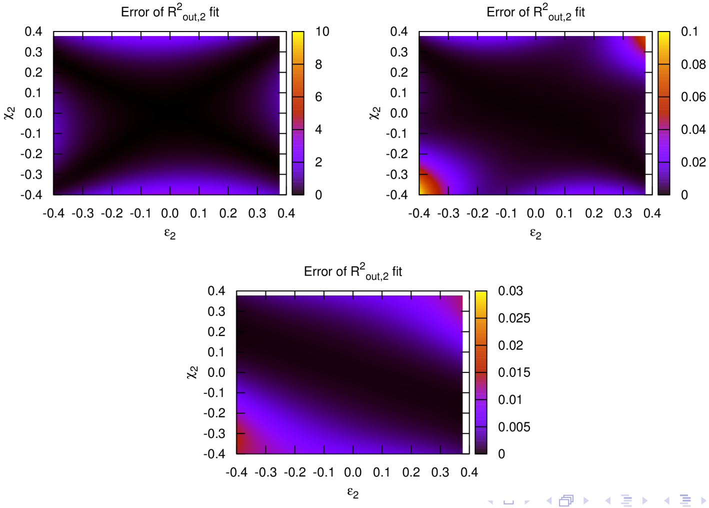
Backup slides – Averaging

Averaging vs. set-to-zero



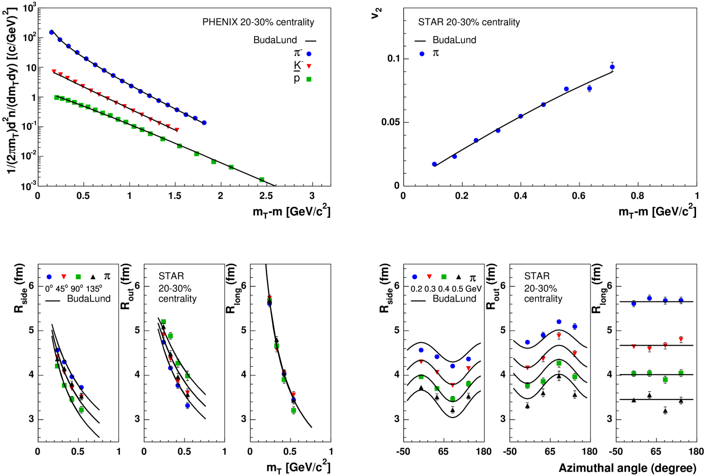
Backup slides – Square of residuals

An example: square of residuals of R^2_{out} with different parametrizations



Earlier results

Fits with elliptical Buda Lund model: Eur.Phys.J. A47 (2011) 58-66



Value of the parameters

Meaning	Sign	Value
Mass of the particle	m	140 MeV
Freeze-out time	τ_0	7 fm/c
Freeze-out temperature	T_0	170 MeV
Temperature-asymmetry parameter	a^2	0.3
Spatial slope parameter	b	-0.1
Transverse size of the source	R	10 fm
Longitudinal size of the source	Z	15 fm
Velocity-space transverse size	H	10 fm/c
Velocity-space longitudinal size	H_z	16 fm/c
Elliptical spatial asymmetry parameter	ϵ_2	0.0
Triangular spatial asymmetry parameter	ϵ_3	0.0
Elliptical velocity-field asymmetry parameter	χ_2	0.0
Triangular velocity-field asymmetry parameter	χ_3	0.0

Usually one anisotropy parameter is varied, and the others are kept zero