# Multipole asymmetries in relativistic hydrodynamics 

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## Hydrodynamics

- Collective behavior observed at RHIC [1]
- Hydro solutions and parametrizations can be applied to measure the initial state of the sQGP
- Famous solutions: Landau, Hwa, Bjorken
- Many new $1+1$ D solutions, a few $1+3 \mathrm{D}$ solutions with spherical, elliptical symmetry
- Parametrizations with spherical, elliptical symmetry


## Buda-Lund model

- Hydro parametrization in final state [2, 3]
- Describe an expanding ellipsoid with a source function

$$
\mathrm{S}(\mathrm{x}, \mathrm{p}) \mathrm{d} \mathrm{x}^{4}=\frac{\mathrm{g}}{(2 \pi)^{3}} \frac{\mathrm{p}^{\nu} \mathrm{d}^{4} \Sigma_{\nu}(\mathrm{x})}{\mathrm{B}(\mathrm{x}, \mathrm{p})+\mathrm{s}_{\mathrm{q}}}
$$

The spatial symmetry is ensured by

$$
\mathrm{s}=\frac{\mathrm{x}^{2}}{\mathrm{X}^{2}}+\frac{\mathrm{y}^{2}}{\mathrm{Y}^{2}}+\frac{\mathrm{z}^{2}}{\mathrm{Z}^{2}} \mathrm{u}_{\mu}=\left(\gamma, \frac{\dot{\mathrm{X}}}{\mathrm{X}} \mathrm{x}, \frac{\dot{\mathrm{Y}}}{\mathrm{Y}} \mathrm{y}, \frac{\dot{\mathrm{Z}}}{\mathrm{Z}^{z}}\right)
$$

The velocity field asymmetry is ellipsoidal too - Successful fit with data $[4,5,6]$

## Higher order anisotropy

- Finite number of nucleons $\Rightarrow$ generalized geometry


Figure 1: Glauber simulation of a $\mathrm{Pb}+\mathrm{Pb}$ collision [7]

- Experimentally observable [8]


Figure 2: $\mathbf{2}^{\text {nd }}$ and $3^{\text {rd }}$ order oscillation in PHENIX experiment.

- Existing solution with arbitrary spatial geometry [9]
- Higher order anisotropies can be described in generalized Buda-Lund model
- Second order case have already been investigated [10]
- Generalization of the
$\triangleright \ldots$ spatial distribution
$\quad \mathrm{s}=\frac{r^{2}}{R^{2}}\left(1+\sum_{\mathrm{n}} \epsilon_{\mathrm{n}} \cos \left(\mathrm{n}\left(\phi-\boldsymbol{\Psi}_{\mathrm{n}}\right)\right)+\frac{\mathrm{r}_{2}^{2}}{\mathrm{Z}^{2}}\right.$
$\triangleright \ldots$ velocity field

$$
\Phi=\frac{r^{2}}{2} H\left(1+\sum_{n} \chi_{n} \cos \left(n\left(\phi-\Psi_{n}\right)\right)+\frac{r_{z}^{2}}{2} H_{z}\right.
$$

- Basically any kind of symmetry can be described in the space-time and in the velocity field


Figure 3: Example flow (left hand side) and density distribution (right

## Observables



- The angle between the reaction planes $\left(\boldsymbol{\Delta}_{2,3}\right)$ should be averaged out
- If the calculation should be fast then the angle can be set to zero


Figure 4: Different radii with different values of $\boldsymbol{\Delta}_{2,3}$

## Invariant spectra

$\checkmark$ Invariant spectra: $\mathbf{N}_{1}(\mathbf{p})=\int \mathbf{d}^{4} \mathbf{x S}(\mathbf{x}, \mathrm{p})$

- The symmetry parameters have no qualitative effect


Figure 5: Azimuthally integrated single-particle $\mathbf{p}_{\mathbf{t}}$ spectra

## Flows

- Flow: $\mathbf{v}_{\mathbf{n}}=\langle\boldsymbol{\operatorname { c o s }}(\mathbf{n} \phi)\rangle_{\mathbf{s}}$
- $\mathbf{n}$-th order flow only depend on $\mathbf{n}$-th order symmetry parameters
- From the flow measurements the value of the parameters cannot be determined



## Oscillating HBT radif

## - HBT radii:

$\triangleright R_{\text {out }}^{2}=\left\langle\left(r_{\text {out }}^{2}-\beta_{\mathbf{t}} \mathbf{t}\right)^{2}\right\rangle-\left\langle\mathbf{r}_{\text {out }}^{2}-\beta_{\mathbf{t}} \mathbf{t}\right\rangle^{2}$
$\triangleright R_{\text {out }}^{2 \mathrm{ou}}=\left\langle\left(r_{\text {side }}^{2 \mathrm{~m}}\right)^{2}\right\rangle-\left\langle r_{\text {side }}^{2}\right)^{2}$

## Placeholder

## Image

Figure 6: Figure caption

## Conclusion

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## Acknowledgments

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