Oscillating HBT from asymmetrical Buda-Lund model

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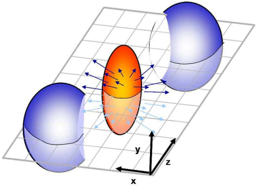
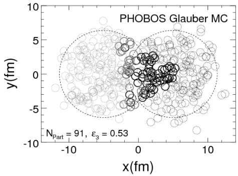
Outline

Introduction - motivation

- sQGP discovered at RHIC and created LHC
- Almost perfect, expanding fluid $\rightarrow$ hydrodynamical approach
- Non-central collision $\rightarrow$ assuming ellipsoidal asymmetry
- Characterize with scaling variable: $s = \frac{r_x^2}{X^2} + \frac{r_y^2}{Y^2} + \frac{r_z^2}{Z^2}$
- But nucleii contain finite number of nucleon
- Generalize the asymmetry $\rightarrow$ higher order anisotropy





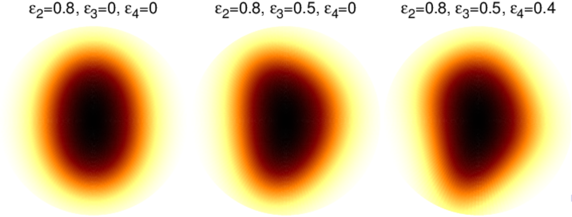
- Can we put it to an exact solution?

Generalization of ellipsoidal symmetry

- Rewrite the scaling variable $s = \frac{r_x^2}{X^2} + \frac{r_y^2}{Y^2} + \frac{r_z^2}{Z^2}$ in cylindrical coordinates
- $s = \frac{r^2}{R^2}(1 + \epsilon_2 \cos(2\varphi))$ where $\frac{1}{R^2} = \frac{1}{X^2} + \frac{1}{Y^2}$ and $\epsilon_2 = \frac{X^2 - Y^2}{X^2 + Y^2}$
- Generalized N-pole symmetry in transverse plane $s = \frac{r^N}{R^N}(1 + \epsilon_N) \cos(N\varphi)$
- Multipole symmetries can be combined in form

$$s = \sum_N \frac{r^N}{R^N}(1 + \epsilon_N \cos(\varphi - \psi_N))$$

- Aligned by Nth order reaction planes ψ_N



New solutions of hydrodynamics

Details: Csanád, Szabó: Phys.Rev.C90,054911 (2014)

- New solution with multipole symmetry

$$s = \sum_N \frac{r^N}{R^N}(1 + \epsilon_N \cos(\varphi - \psi_N)) + \frac{z^N}{Z^N}$$

$$u^\mu = \gamma \left(1, \frac{\dot{R}}{R} r \cos\varphi, \frac{\dot{R}}{R} r \sin\varphi, \frac{\dot{R}}{R} z\right)$$

$$T = T_f \left(\frac{\tau_f}{\tau}\right)^{\frac{3}{\kappa}} \frac{1}{\nu(s)} \quad \text{choose Gaussian:} \quad \nu(s) = e^{bs}$$

- Observed higher order harmonics:
  $S(x,p) \propto exp\left(\frac{p_\mu u^\mu(x)}{T(x)}\right) \delta(\tau, \tau_f) \frac{p_\mu u^\mu}{u_0}$
- Momentum distribution $N_1(p)$ and anisotropies $v_n(p_t)$ can be yielded
- Succesful fit!

What missing from the solution

- Real 3+1D solution
- Take into account the general spatial symmetry
- Still use Hubble-type velocity field
- Collective behaviour demand to generalized this
- Use of constant κ
- These problem can be solved in a hydrodynamical model

Ellipsoidal Buda-Lund model

Csanád, Csörgő, Lorstad Nucl.Phys.A742, 80-94 (2004)

- Final state parametrization
- Ellipsoidal symmetry in space and in velocity field (Hubble-type)

$$s = \frac{r_x^2}{2X^2} + \frac{r_y^2}{2Y^2} + \frac{r_z^2}{2Z^2} \qquad u_\mu = \left(\gamma, r_x\frac{\dot{X}}{X}, r_x\frac{\dot{Y}}{Y}, r_z\frac{\dot{Z}}{Z}\right)$$

- Thermal distribution is influented by spatial geometry

$$\frac{1}{T} = \frac{1}{T_0}\left(1 + a^2 s\right)$$

- Source function of the model

$$S(x,p)d^4x = \frac{g}{(2\pi)^3}\frac{p_\mu u^\mu H(\tau) d^4x}{e^{\frac{p_\mu u^\mu - \mu}{T}} - s_q}$$

$H(\tau)$ is a Gaussian function centered to τ_0 with width $\Delta\tau^2$

Observables from ellipsoidal Buda-Lund model

- Saddle-point (SP) approximation → source function can be integrated analitically

$$S(x,p)d^4x = \frac{g}{(2\pi)^3}\frac{p_\mu u^\mu H(\tau)d^4x}{e^{\frac{p_\mu u^\mu - \mu}{T}} + s_q} e^{R^{-2}_{\mu\nu}(x-x_s)^\mu (x-x_s)^\nu}$$

- $R_{\mu\nu} = \partial_\mu \partial_\nu (-\ln S_0(x,p))|_s$, where $S_0(x,p) = \frac{H(\tau)}{B(x,p)+s_q}$
- The SP is definied by $\partial_\mu(-\ln S_0(x_s,p)) = 0$
- $\int d^4x S(x,p) = N_1(p)$ can be calculated
- In transverse momentum space $N_1(p_t)$ distribution and $v_2(p_t)$ flow can be yielded

$$v_n = \frac{\int_0^{2\pi} d\alpha N_1(p)\cos(n\alpha)}{\int_0^{2\pi} d\alpha N_1(p)} = \frac{I_n(w)}{I_0(w)}$$

- In this model v_{2n+1} flows are vanishing

Generalized model

- Change to cylindrical coordinates: $(x, y, z) \to (r, \varphi, r_z)$
- Ellipsoidal scaling variable: $s = \frac{r^2}{R^2}(1 + \epsilon_2 \cos(2\varphi)) + \frac{r_z^2}{Z^2}$
- Modify this with ϵ_3 to describe the triangular symmetry
- Trianular scale function:

$$s = \frac{r^2}{R^2}(1 + \epsilon_2 \cos(2\varphi)) + \frac{r^3}{R^3}\epsilon_3 \cos(3\varphi) + \frac{r_z^2}{Z^2}$$

- The velocity field should be generalized too!
- Calculate from a potential:

$$\Phi = \frac{r^2}{2H}(1 + \chi_2 \cos(2\varphi)) + \frac{r_z}{H_z}$$

- Modify the potential:

$$\Phi = \frac{r^2}{2H}(1 + \chi_2 \cos(2\varphi)) + \frac{r^3}{3R^2}\chi_3 \cos(3\varphi) + \frac{r_z}{H_z}$$

- The velocity fied can be calculated as $u_\mu = (\gamma, \partial_x\Phi, \partial_y\Phi, \partial_z\Phi)$

The velocity field

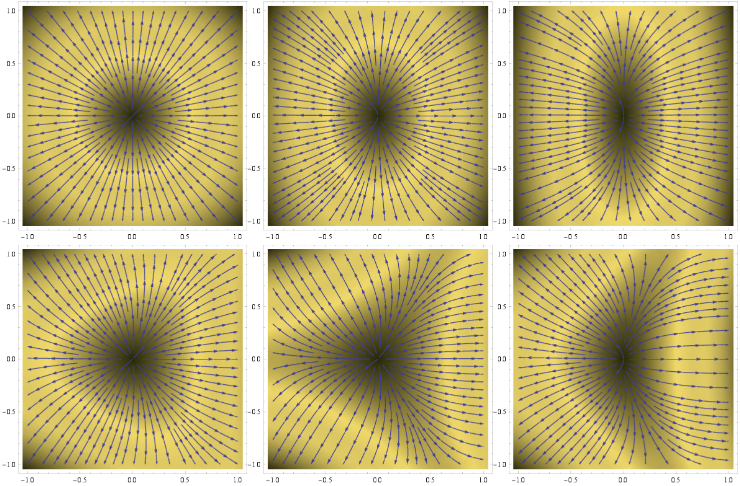


Figure : 1. $\chi_2 = \chi_3 = 0$, 2. $\chi_2 = 0.2$, 3. $\chi_2 = 0.3$,
4. $\chi_3 = 0.3$, 5. $\chi_3 = 0.4$, 6. $\chi_2 = 0.3$, $\chi_3 = 0.3$

Observables from the generalized model

- Integrate the source function with the new scale function and the new velocity field
- SP approximation cannot be used as early
- Numerical calculation in progress but another approximation can be applied
- Let assume $\left(\frac{r_i}{H_i}\right)^n$ are vanishing if n>2
- The SP integral can be calculate analytically
- Yield the invariant momentum distribution, elliptical, triangular flow and HBT radii

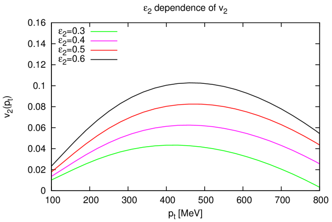
Parameter dependence of the flows

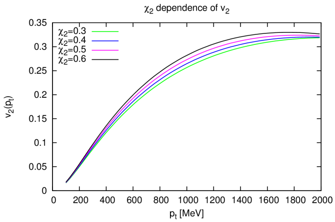
Expected:

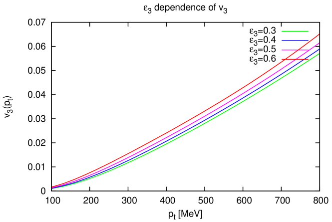
- v_2 depend on ϵ_2 and χ_2
- v_2 not depend on ϵ_3 and χ_3

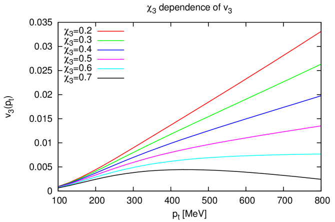
v_2 depend on ϵ_2 and χ_2

v_3 not depend on ϵ_2 and χ_2



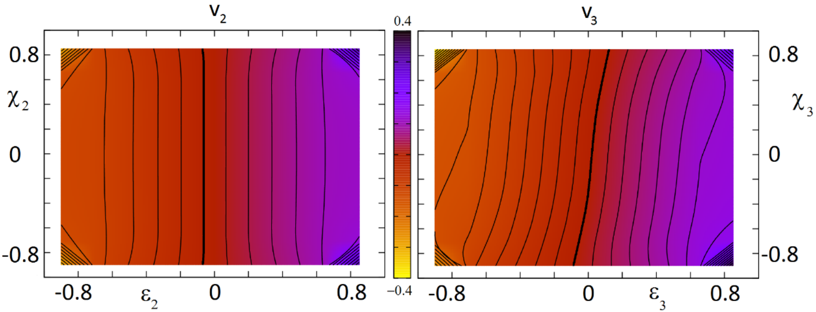






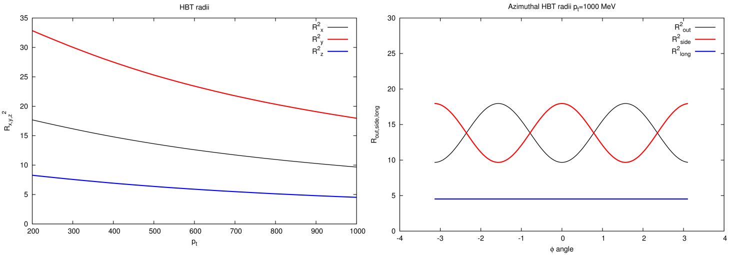
Combination of the parameter

- v_i is determined by ϵ_i and χ_i, especially true for v_3
- There is no such a solutions found yet



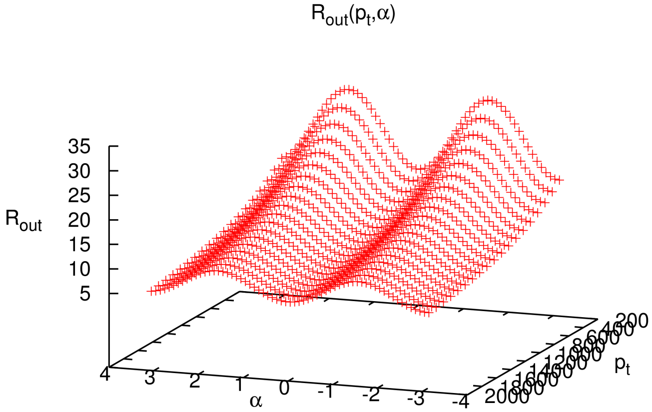
Azimuthally sensitive HBT radii

- Explicit HBT radii can be yielded: $R_i^2 = \dfrac{X_i^2}{2\left(\frac{E}{T_0}a^2 - b\right)}$
- Not depend on asymmetry parameters or transverse angle
- Rotate system to *out* − *side* − *long* system with angle φ



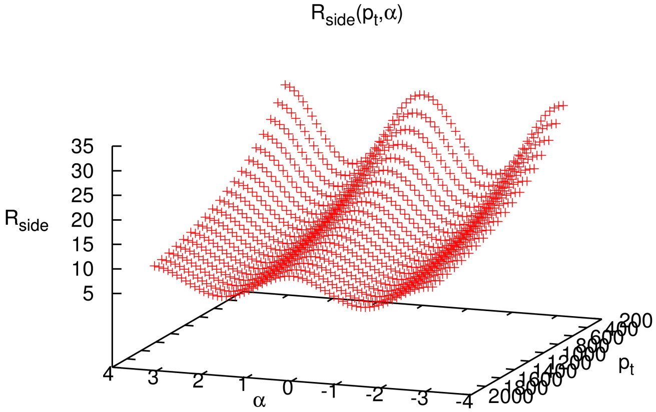
Azimuthally sensitive HBT radii in 3D

$$R_{out}^2 = \frac{R_x^2 + R_y^2}{2} - \frac{R_y^2 - R_x^2}{2}\cos(2\varphi)$$



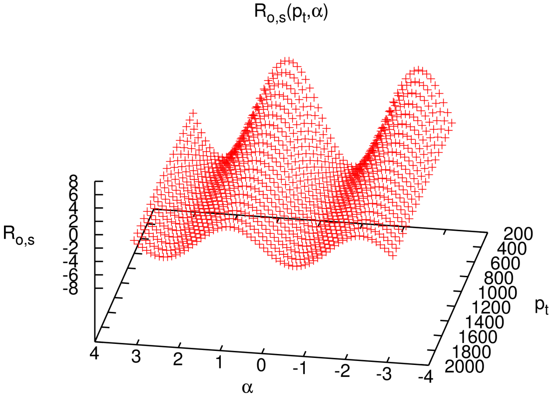
Azimuthally sensitive HBT radii in 3D

$$R_{side}^2 = \frac{R_x^2 + R_y^2}{2} + \frac{R_y^2 - R_x^2}{2}\cos(2\varphi)$$



Azimuthally sensitive HBT radii in 3D

$$R_{o,s} = \frac{R_y^2 - R_x^2}{2} \sin(2\varphi)$$



Summary and outlook

- Generalized asymmetry implemented to a hydrodynamical solution
- Compared with data succesfully!
- Still Hubble-type velocity profile
- Generalized asymmetry implemented to a hydrodynamical model
- Extend the symmetry to the velocity field
- With a harsh approxmation $N_1(p_t), v_2(p_t), v_3(p_t)$ yielded
- More precise with numerical analyzis (in progress)
- ϵ_2, ϵ_3 and χ_2, χ_3 dependence is important

Thank you for your attention!