# Description of oscillating HBT radii in Buda-Lund hydrodynamical model

Máté Csanád, Sándor Lökös, Boris Tomášik and Tamás Csörgő

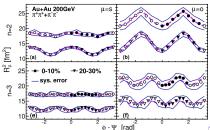
Zimányi School, 2015

#### Introduction

- ullet QGP behaves like perfect fluid o hydro description
- ullet Finite number of nucleons o generalized geometry is nescessary
- Generalize the space-time and the velocity field distribution
- Higher order flows can be investigated
- ullet HBT radii have  $\cos(n\phi)$  dependences in the respective reaction plane
- These can be studied experimentally:

Nucl.Phys. A904-905 (2013) 439c-442c

Phys.Rev.Lett. 112 (2014) 22, 222301



#### The Buda-Lund model

Phys.Rev. C54 (1996) 1390 and Nucl. Phys. A742 (2004) 80-94

- Hydro-model:  $S(x,p)=\frac{g}{(2\pi)^3}\frac{p^{\nu}d^{4}\Sigma_{\nu}(x)}{B(x,p)+s_q}$  where  $B(x,p)=\exp\left[\frac{p^{\nu}u_{\nu}(x)-\mu(x)}{T(x)}\right]$  is the Boltzmann phase-space distribution and the  $p^{\nu}d^{4}\Sigma_{\nu}(x)=p^{\nu}u_{\nu}H(\tau)d^{4}x$
- Spatial elliptical asymmetry is ensured by the scaling variable

$$s = \frac{r_x^2}{2X^2} + \frac{r_y^2}{2Y^2} + \frac{r_z^2}{2Z^2} \rightarrow \frac{r^2}{2R^2} (1 + \epsilon_2 \cos(2\phi)) + \frac{r_z^2}{2Z^2}$$

The asymmetry in the velocity field is also elliptical

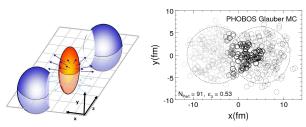
$$u_{\mu} = \left(\gamma, r_{x} \frac{\dot{X}}{X}, r_{y} \frac{\dot{Y}}{Y}, r_{z} \frac{\dot{Z}}{Z}\right) \rightarrow (\gamma, rH(1 + \chi_{2}) \cos \phi, rH(1 - \chi_{2}) \sin \phi, H_{z}r_{z})$$

#### Generalization of the model 1.

- The spatial asymmetry is described by the scaling variable
- General *n*-pole spatial asymmetry (elliptical case: n = 2):

$$s = \frac{r^2}{2R^2} \left( 1 + \sum_n \epsilon_n \cos(n(\phi - \Psi_n)) \right) + \frac{r_z^2}{2Z^2}$$

ullet  $\Psi_n$  is the angle of the *n*-th order reaction plane

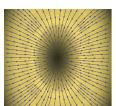


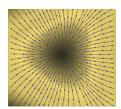
### Generalization of the model II.

- Derive the velocity field from a potential:  $u_{\mu} = \gamma(1, \partial_x \Phi, \partial_y \Phi, \partial_z \Phi)$
- General *n*-pole asymmetrical potential (elliptical case: n = 2):

$$\Phi = Hr^2 \left( 1 + \sum_n \chi_n \cos(n(\phi - \Psi_n)) \right) + H_z r_z^2$$

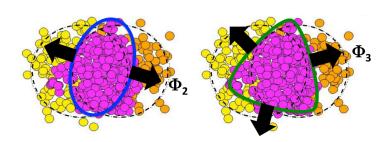
• There is multipole solution: Phys.Rev.C90,054911 (2014) based on HeavylonPhys.A21:73-84,2004





#### Observables at freeze-out

- Invariant transverse momentum distribution, flows, azimuthally sensitive HBT radii
- All asymmetries are investigated in their respective reaction plane
- Rotate the system to the second / third order plane and average on the angle of the third / second order plane



# Averaging on event planes

• The spatial asymmetry:

$$s = \frac{r^2}{R^2} (1 + \epsilon_2 \cos(2\phi - \Psi_2) + \epsilon_3 \cos(3\phi - \Psi_3)) + \frac{r_z^2}{Z^2}$$

• If we rotate the system to the second order event plane:

$$s = \frac{r^2}{R^2}(1 + \epsilon_2 \cos(2\phi) + \epsilon_3 \cos(3\phi - \Delta \Psi_{2,3})) + \frac{r_z^2}{Z^2}$$

• If we rotate the system to the third order event plane:

$$s = \frac{r^2}{R^2}(1+\epsilon_2\cos(2\phi+\Delta\Psi_{2,3})+\epsilon_3\cos(3\phi)) + \frac{r_z^2}{Z^2}$$

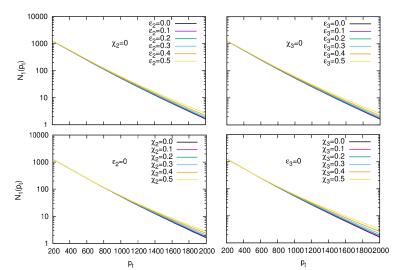
where  $\Delta\Psi_{2,3}=\Psi_3-\Psi_2$ .

- The method is the same in the case of the velocity field.
- Set to zero the parameter directly or averaging on  $\Delta \Psi_{2,3}$ ?



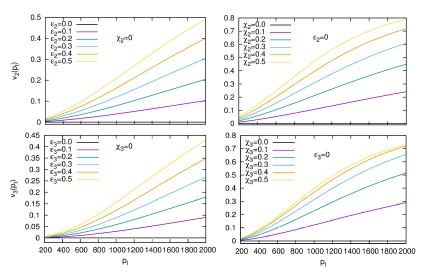
#### Invariant momentum distribution

Significant change could be at high  $p_t$ , the log slope is not affected strongly



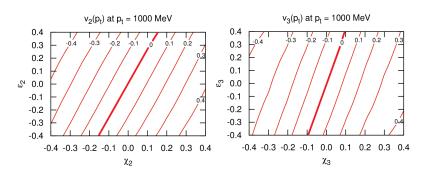
#### Flows

### Elliptic and triangular flows are affected by their own asymmetry parameters



# Mixing of parameters

- The parameters affect the flows together
- The generalization of velocity field is nescessary



#### HBT radii

Calculate in the out — side — long system

$$R_{
m out}^2=\langle r_{
m out}^2
angle-\langle r_{
m out}
angle^2$$
 and  $R_{
m side}^2=\langle r_{
m side}^2
angle-\langle r_{
m side}
angle^2$ 

where 
$$r_{\text{out}} = r \cos(\phi - \alpha) - \beta_t t$$
 and  $r_{\text{side}} = r \sin(\phi - \alpha)$   
 $\rightarrow$  C. J. Plumberg, C. Shen, U. W. Heinz Phys.Rev. C88 (2013) 044914

- There can be higher order parts
  - ightarrow B. Tomášik and U. A. Wiedemann, in *QGP3*, pp. 715–777.
- We use the following parameterization in
  - elliptical case:

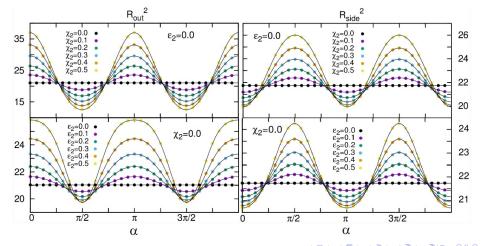
$$R_{\mathrm{out}}^2 = R_{\mathrm{out},0}^2 + R_{\mathrm{out},2}^2 \cos(2\alpha) + + R_{\mathrm{out},4}^2 \cos(4\alpha) + R_{\mathrm{out},6}^2 \cos(6\alpha)$$

- triangular case:  $R^2 - R^2 + R^2 \cos(3\alpha) + R^2 \cos(6\alpha) + R^2$
- $R_{\mathsf{out}}^2 = R_{\mathsf{out},0}^2 + R_{\mathsf{out},3}^2 \cos(3\alpha) + R_{\mathsf{out},6}^2 \cos(6\alpha) + R_{\mathsf{out},9}^2 \cos(9\alpha)$
- Similar to the  $R_{\text{side}}^2$



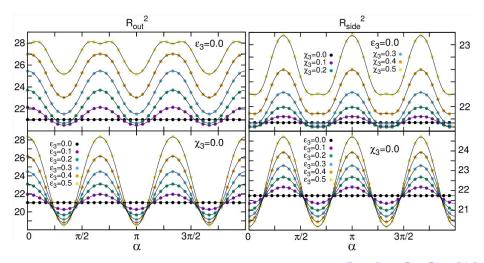
# Results of the parametrization – Second order case

This case already have investigated: Eur.Phys.J.A37:111-119,2008 Mainly  $\cos(2\phi)$  behavior but higher order oscillations are also present



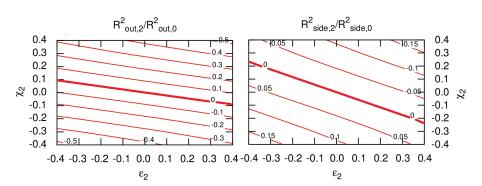
# Results of the parametrization – Third order case

Mainly  $cos(3\phi)$  behavior but higher order oscillations are also present



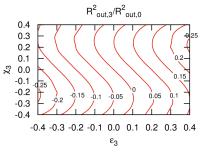
# Mixing of the parameters

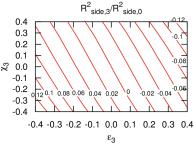
The dependence of the amplitudes of the  $R_{\text{out}}^2$  and  $R_{\text{side}}^2$  in the second order case



# Mixing of the parameters

The dependence of the amplitudes of the  $R_{\text{out}}^2$  and  $R_{\text{side}}^2$  in the third order case





#### Conclusions

- Generalization of the spatial and the velocity field distribution is done
- The averaging between the different event plane is nescesary
- Higher order oscillation can be observed in HBT radii
- Absolute value of the azimuthal HBT radii depend on asymmetries
- The spatial and velocity field anisotropies both influence the  $v_n$  coefficient and the HBT radii
- The asymmetry parameters can be disentangle from the flows and the amplitudes

# Thank you for your attention!

### Value of the parameters

Meaning	Sign	Value
Mass of the particle	m	140 MeV
Freeze-out time	$   au_0  $	7 fm/c
Freeze-out temperature	$T_0$	170 MeV
Temperature-asymmetry parameter	$a^2$	0.3
Spatial slope parameter	Ь	-0.1
Transverse size of the source	R	10 fm
Longitudinal size of the source	Z	15 fm
Velocity-space transverse size	Н	10 c/fm
Velocity-space longitudinal size	$H_z$	16 c/fm
Elliptical spatial asymmetry parameter	$\epsilon_2$	0.0
Triangular spatial asymmetry parameter	$\epsilon_3$	0.0
Elliptical velocity-field asymmetry parameter	$\chi_2$	0.0
Triangular velocity-field asymmetry parameter	$\chi_3$	0.0

Usually one anisotropy parameter is varied, and the others are kept zero

## About the spectra

Plot the  $N_1$  with non zero coefficient divide by  $N_1$  with zero coefficient

